Intermediate Econometrics — Instrumental Variables

Practice Exam A

Time: 50 minutes

Question 1. Reduced Form, First Stage, and ITT (15 points)

A job training program randomly assigns individuals to receive an offer of training.
Let:

- $Z_i \in \{0,1\}$: indicator for being offered training

- $D_i \in \{0,1\}$: indicator for actually receiving training

- Y_i: post-program earnings

You observe the following sample means:

Group	$E[D_i]$	$E[Y_i]$
$Z = 1$	0.60	22,000
$Z = 0$	0.05	20,500

(a) Define the **first stage**, **reduced form**, and **ITT effect**.

(b) Compute each of them using the table.

(c) Explain why the reduced form has a causal interpretation.

Answer (A1)

(a) Definitions

- First stage:
$$E[D_i \mid Z_i = 1] - E[D_i \mid Z_i = 0].$$

- Reduced form (ITT on outcomes):
$$E[Y_i \mid Z_i = 1] - E[Y_i \mid Z_i = 0].$$

- ITT effect: identical to the reduced form in this setting; it is the causal effect of being offered treatment.

(b) Calculations

- First stage:
$$0.60 - 0.05 = 0.55.$$

- Reduced form / ITT:
$$22{,}000 - 20{,}500 = 1{,}500.$$

(c) Explanation   Because Z_i is randomly assigned, it is independent of potential outcomes. Therefore, differences in outcomes by Z_i identify the causal effect of assignment to treatment (the ITT effect), even though compliance is imperfect.

Question 2. Wald Estimator and LATE (15 points)

(a) Compute the Wald estimator using the numbers above.

(b) State precisely what causal parameter the Wald estimator identifies.

(c) Under what assumptions does this interpretation hold?

Answer (A2)

(a) Wald estimator
$$\hat{\beta}_{IV} = \frac{\text{Reduced Form}}{\text{First Stage}} = \frac{1500}{0.55} \approx 2727.$$

(b) Interpretation   The Wald estimator identifies the **Local Average Treatment Effect (LATE)**: the average causal effect of training on earnings for *compliers*, i.e., individuals who receive training if and only if they are offered it.

(c) Required assumptions

1. Independence: $Z_i \perp (Y_i(0), Y_i(1), D_i(0), D_i(1))$

2. Exclusion restriction: Z_i affects Y_i only through D_i

3. First-stage relevance: $P(D_i = 1 \mid Z_i = 1) \neq P(D_i = 1 \mid Z_i = 0)$

4. Monotonicity: no defiers $(D_{1i} \geq D_{0i})$

Question 3. Compliance Types (10 points)

(a) Define compliers, always-takers, never-takers, and defiers.

(b) Which groups contribute to the IV estimate? Why?

Answer (A3)

(a) Definitions

- Compliers: $D_{1i} = 1, \; D_{0i} = 0$

- Always-takers: $D_{1i} = D_{0i} = 1$

- Never-takers: $D_{1i} = D_{0i} = 0$

- Defiers: $D_{1i} = 0, \; D_{0i} = 1$

(b) Contribution to IV

Only compliers contribute to the IV estimate because:

- The treatment status of always-takers and never-takers does not change with the instrument.

- IV relies on variation in D_i induced by Z_i.

Practice Exam B

Question 1. Conceptual (15 points)

Explain clearly the difference between:

1. Independence assumption

2. Exclusion restriction

Use the draft lottery as an example.

Answer (B1)

- **Independence**: the instrument is as good as randomly assigned:

$$Z_i \perp (Y_i(d), D_i(z)).$$

In the draft lottery, lottery numbers are randomly assigned.

- **Exclusion restriction**: the instrument affects outcomes only through the endogenous variable:

$$Y_i(d, 1) = Y_i(d, 0).$$

Draft eligibility must affect earnings only through military service.

Random assignment does not guarantee exclusion: draft eligibility might affect schooling decisions, violating exclusion even if independence holds.

Question 2. Failure of Monotonicity (10 points)

Explain why the presence of defiers breaks the link between the reduced form and LATE.

Answer (B2)

Without monotonicity, the reduced form equals:

$$E[(Y_{1i} - Y_{0i})(D_{1i} - D_{0i})],$$

which combines positive effects for compliers and negative-weighted effects for defiers. These can cancel out, so the reduced form may be zero even when treatment effects are positive for everyone.

Practice Exam C

Question 1. 2SLS with Covariates (15 points)

Explain why, in covariate-adjusted 2SLS, the instrument is residualized with respect to covariates before being used.

Answer (C1)

Residualizing the instrument removes variation correlated with exogenous covariates. This ensures that identification comes only from variation in the instrument that is orthogonal to controls, consistent with the Frisch–Waugh–Lovell theorem. Intuitively, 2SLS isolates quasi-experimental variation generated by the instrument.

Question 2. Interpretation (10 points)

Explain the statement:

"2SLS retains only the variation in D_i generated by the instrument."

Answer (C2)

2SLS replaces D_i with its fitted values from the first stage, which depend only on Z_i (and controls). This discards endogenous variation in D_i and keeps only variation induced by the instrument, which is assumed to be exogenous.